

A) Distinct Roots $\rightarrow$ Real $\rightarrow D > 0$

B) Eq^l Root $\rightarrow D = 0$

C) Imaginary Root $\Rightarrow D < 0$

D) Reciprocal Root $\rightarrow \alpha, \frac{1}{\alpha} \rightarrow \text{Product} = 1$

E) Both Roots +ve $\rightarrow D \geq 0, \alpha + \beta = +ve, \alpha * \beta = +ve$

F) Exactly 1 Root +ve $D > 0, \alpha + \beta < 0, \alpha * \beta < 0$

G) At least one Root +ve $\rightarrow$ $\begin{cases} \text{Exactly 1 Root +ve} \\ \text{Both Root +ve} \end{cases} \rightarrow \boxed{E \cup F}$ Boundary Value Check

Q4 Roots of

$$(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 = 0$$

are eq^l then P.T. $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$$a, b, c, d \in \mathbb{R} \quad a, b, c \neq 0$$

$$M1 \quad \boxed{1) = 0}$$

$$(a^2x^2 - 2abx + b^2) + (b^2x^2 - 2bcx + c^2) + (c^2x^2 - 2cdx + d^2) = 0$$

$$\Rightarrow \underbrace{(ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0}$$

$$ax - b = 0 \text{ \& } bx - c = 0 \text{ \& } cx - d = 0$$

$$x = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Q α, β are Roots of $ax^2 + bx + c = 0$

Very Basic $\alpha + k, \beta + k$ are Roots of $Ax^2 + Bx + C = 0$

$$\text{then P.T. } \frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$$

$$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \rightarrow |\alpha - \beta| = \frac{\sqrt{b^2 - 4ac}}{|a|}$$

$$Ax^2 + Bx + C = 0 \begin{matrix} \nearrow \alpha + k \\ \searrow \beta + k \end{matrix} \rightarrow |\alpha + k - \beta - k| = \frac{\sqrt{B^2 - 4AC}}{|A|}$$

$$\text{LHS = Same} \quad \frac{\sqrt{B^2 - 4AC}}{|A|}$$

Put Sq^r & Prove

Q If $x^2 + ax + b = 0$ has Roots $\sec^2 \theta$ & $\csc^2 \theta$ then find $a+b=?$

So here

$$\sec^2 \theta + \csc^2 \theta = \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$\boxed{\sec^2 \theta + \csc^2 \theta = \sec^2 \theta \cdot \csc^2 \theta}$$

$$\downarrow \quad \quad \quad \downarrow$$

$$\text{SOR} = \text{POR}$$

$$-\frac{a}{1} = \frac{b}{1} \Rightarrow a+b=0$$

Q If α, β are roots of $(x-a)(x-b)+c=0$
Basic then find Roots of $\boxed{(x-\alpha)(x-\beta)-c=0}$

Basic $\rightarrow ax^2+bx+c=0$ has Root α, β if
 $ax^2+bx+c = a(x-\alpha)(x-\beta)$

$$(x-a)(x-b)+c=0 \xrightarrow{\alpha, \beta \text{ are roots}} (x-\alpha)(x-\beta)+c=0$$

$$(x-a)(x-b)+c = 1 \cdot (x-\alpha)(x-\beta)+c$$

$$\frac{(x-a)(x-b)}{\text{Roots } a, b} = \frac{(x-\alpha)(x-\beta)+c}{\text{Roots } \alpha, \beta}$$

$$\text{Roots } -a, b$$

Q If $x^2 + x + 1 = 0$ then value of:

$$\left(x + \frac{1}{x}\right)^2 + 2\left(x^2 + \frac{1}{x^2}\right)^2 + 3\left(x^3 + \frac{1}{x^3}\right)^2 + 5\left(x^5 + \frac{1}{x^5}\right)^2$$

$$(-1)^2 + 2(-1)^2 + 3(2)^2 + 5(-1)^2 = ?$$

$$1) \quad x^2 + x + 1 = 0 \Rightarrow x + 1 + \frac{1}{x} = 0 \Rightarrow \boxed{x + \frac{1}{x} = -1}$$

$$2) \quad \left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2 = (-1)^2 - 2 = -1$$

$$3) \quad \left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ = (-1)^3 - 3(-1) = -1 + 3 = 2$$

$$(4) \quad x^5 + \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right) = \underline{\underline{(-1)(2)}} + \left(x + \frac{1}{x}\right)$$

$$(-1)(2) = \left(x^5 + \frac{1}{x^5}\right) + (-1)$$

$$5(-1)^2 + 3(2)^2 + 2(-1)^2 + (-1)^2$$

$$5 + 12 + 2 + 1 = 20$$

Q If $(h-c)x^2 + (c-a)x + (a-h) = 0$
has eq^l Roots then Rel. betⁿ a, b, c.
Cyclic $\rightarrow \alpha = 1 = \beta$.

$$POR = 1$$

$$\frac{a-b}{h-c} = 0 \Rightarrow a-b = h-c$$

$$\Rightarrow a+c = 2b$$

Q. If $\alpha^2 + 3 = 5\alpha$ & $\beta^2 = 5\beta - 3$
find $\alpha + \beta = ?$

$$\alpha^2 - 5\alpha + 3 = 0 \quad \& \quad \beta^2 - 5\beta + 3 = 0$$

$$\alpha^2 - 5\alpha + 3 = 0 \xrightarrow{\alpha} \beta \quad \alpha + \beta = -\frac{(-5)}{1} = 5$$

Q If α, β are Roots $\boxed{x^2 - 2x + 5 = 0}$
then form a Q Eqⁿ whose Roots are
 $= \boxed{\alpha^3 + \alpha^2 - \alpha + 22}$ & $\boxed{\beta^3 + 4\beta^2 - 7\beta + 35}$?

$$x^2 - 2x + 5 = 0 \xrightarrow{\alpha} \alpha^2 - 2\alpha + 5 = 0$$

$$\xrightarrow{\beta} \beta^2 - 2\beta + 5 = 0$$

$$\begin{array}{r} \alpha^2 - 2\alpha + 5 \overline{) \beta^3 + \alpha^2 - \alpha + 22} \quad (\alpha + 3) \\ \underline{\alpha^3 - 2\alpha^2 + 5\alpha} \\ 3\alpha^2 - 6\alpha + 22 \\ \underline{3\alpha^2 - 6\alpha + 15} \\ +7 \end{array}$$

$$\alpha^3 + \alpha^2 - \alpha + 22 = (\alpha^2 - 2\alpha + 5)(\alpha + 3) + 7$$

$$= 7$$

$$\begin{array}{r}
 (B^2 - 2B + 5)B^3 + 4B^2 - 7B + 35 \quad (B+6) \\
 \underline{B^3 - 2B^2 + 5B} \\
 6B^2 - 12B + 35 \\
 \underline{6B^2 - 12B + 36} \\
 5
 \end{array}$$

$$B^3 + 4B^2 - 7B + 35 = (B^2 - 2B + 5)(B+6) + 5$$

$$\begin{aligned}
 \text{Eqn } (x-7)(x-5) &= 0 \\
 x^2 - 12x + 35 &= 0
 \end{aligned}$$

Q

$$a^2(-a^3 + b^3) = 3abc$$

find "and" for which Q Eqn.

$ax^2 + bx + c = 0$ has (A) one root sq of
other (B) one root n^{th} power of other

$$ax^2 + bx + c = 0 \xrightarrow{x^2} x + \frac{b}{a}x + \frac{c}{a} = 0 \quad \left| \begin{array}{l} x + x^2 = -\frac{b}{a} \\ x \cdot x^3 = \frac{c}{a} \\ x^3 = \frac{c}{a} \end{array} \right.$$

$$(x + x^2)^3 = -\frac{b^3}{a^3}$$

$$x^3 + 3x^4 + 3x^5 + x^6 = -\frac{b^3}{a^3}$$

$$\frac{c}{a} + 3x^3(x + x^2) + \frac{c^2}{a^2} = -\frac{b^3}{a^3}$$

$$\frac{c}{a} - \frac{3bc}{a^2} + \frac{c^2}{a^2} = -\frac{b^3}{a^3}$$

$$a^2(-3abc + a^2 + b^3) = 0$$

$$ax^2 + bx + c = 0 \quad \begin{matrix} \nearrow x \\ \searrow x^n \end{matrix}$$

$$x + x^n = -\frac{b}{a} \quad | \quad x \cdot x^n = \frac{c}{a}$$

$$x = \left(\frac{c}{a}\right)^{\frac{1}{n+1}}$$

$$\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -\frac{b}{a}$$

$$ax^{\frac{1}{n+1}} + ax^{\frac{n}{n+1}} = -b$$

$$a^{1-\frac{1}{n+1}} \cdot \left(\frac{c}{a}\right)^{\frac{1}{n+1}} + a^{1-\frac{n}{n+1}} \cdot \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -b$$

$$a^{\frac{n}{n+1}} \cdot \left(\frac{c}{a}\right)^{\frac{1}{n+1}} + a^{\frac{1}{n+1}} \cdot \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -b$$

$$\boxed{\left(a^n c\right)^{\frac{1}{n+1}} + \left(a \cdot c^n\right)^{\frac{1}{n+1}} = -b}$$

Q If One Root of $x^2 - x - k = 0$ is sqⁿ of other then k?

$$a=1, b=-1, c=-k$$

↓

$$a^2(c+ac^2+b^3) = 3abc$$

$$x^2 - x - k = 0 \begin{matrix} \nearrow x \\ \searrow x^2 \end{matrix}$$

$$x + x^2 = 1 \mid x^3 = -k$$

$$1^2 \cdot k + 1 \cdot (-k)^2 + (-1)^3 = 3 \times 1 \times -1 \times -k$$

$$-k + k^2 - 1 = 3k$$

$$k^2 - 4k - 1 = 0$$

$$k = \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$k = 2 + \sqrt{5}, 2 - \sqrt{5}$$

Q If one Root of $x^2+px+q=0$ is S.I. of other then S.I.

$$p^3+q^2+q(1-3p)=0.$$

$$\downarrow$$

$$a^2c+ac^2+b^3=3abc$$

$$a=1, b=p, c=q$$

$$1^2 \cdot q + 1 \cdot q^2 + p^3 = 3 \cdot p \cdot q$$

$$p^3+q^2+q-3pq=0$$

$$p^3+q^2+q(1-3p)=0$$

Q Find condⁿ when one root is k times of other

$$ax^2 + bx + c = 0 \begin{cases} \alpha \\ k\alpha \end{cases}$$

$$\alpha + k\alpha = -\frac{b}{a}$$

$$\alpha(1+k) = -\frac{b}{a}$$

$$\alpha = \frac{-b}{a(1+k)}$$

$$\alpha^2 k = \frac{c}{a}$$

$$\alpha^2 = \frac{c}{ak}$$

$$\frac{b^2}{a^2(1+k)^2} = \frac{c}{ak}$$

$$\boxed{\frac{b^2}{ac} = \frac{(1+k)^2}{k}}$$

Required condⁿ →

Q If α, β are roots

$$\text{of } 5x^2 + mx + 12 = 0$$

Which are in Ratio 2:3.

then $m = ?$

$$5x^2 + mx + 12 = 0 \begin{cases} \alpha \\ \frac{2}{3}\alpha = \beta \end{cases}$$

$$\frac{b^2}{ac} = \frac{(k+1)^2}{k}$$

$$\Rightarrow \frac{m^2}{60} = \frac{\left(\frac{2}{3} + 1\right)^2}{2} = \frac{25}{8} \times \frac{2}{2}$$

$$m^2 = 60 \times \frac{25}{8}$$

$$\underline{m = \pm 5\sqrt{10}}$$

Q Find value of a for which one root of quad.

$$(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0 \xrightarrow{K=2}$$

is twice as large as other. $K=2$

$$\frac{b^2}{ac} = \frac{(K+1)^2}{K} \rightarrow \frac{(3a-1)^2}{(a^2-5a+3)2} = \frac{(2+1)^2}{2}$$

$$9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$39a = 26$$

$$a = \frac{2}{3}$$

Roots Under Particular Condⁿ

$$ax^2 + bx + c = 0$$

1) $\boxed{b=0}$

$$ax^2 + c = 0$$

$$x = \pm \sqrt{\frac{c}{a}}$$

Roots opp sign

(2) $c=0$

$$ax^2 + bx = 0$$

$$x(ax + b) = 0$$

$$x = 0, x = -\frac{b}{a}$$

One Root = 0

(3) $b=c=0$

$$ax^2 = 0$$

$$x = 0, 0$$

Both Root = 0

(4) $a = -c$

$$\alpha \cdot \beta = \frac{c}{a} = 1$$

$$\beta = \frac{1}{\alpha}$$

Reciprocal
Roots

(5) a, b, c Same Sign

$$\alpha + \beta = -\frac{b}{a} = -ve$$

$$\alpha \cdot \beta = +ve$$

$$\alpha, \beta = -ve$$

(6) a, b, c alternative
opp sign.

$$a+ \quad b- \quad c+$$

$$\alpha + \beta = +ve$$

$$\alpha \cdot \beta = +ve$$

$$\alpha, \beta = +ve$$

LOGARITHM & QUADRATIC EQUATION

1. The value of the expression $\frac{1}{\log_4(18)} + \frac{1}{2\log_6(3) + \log_6(2)} + \frac{5}{\log_3(18)}$, is

(A) odd

(B) an irrational

(C) even composite

(D) twin prime with 5

$$\log_{18} 4 + \log_{18} 6 + 5 \log_{18} 3 = 3.$$

LOGARITHM & QUADRATIC EQUATION

2. Given that $\frac{1}{\log_7 2} + \frac{1}{\log_9 4} = x$, then which of the following will divide $(4)^x$?
- (A) 3 (B) 7 (C) 9 (D) 21

LOGARITHM & QUADRATIC EQUATION

3. If $\log_5 (3^x - 4^y) = 3$ and $3^{\frac{x}{2}} - 2^y = 5$, then $\frac{x}{y}$ is equal to

(A) $\frac{2(\log_2 5) - 2}{1 + \log_2 5}$

(B) $\frac{(\log_3 5) + 2}{1 + \log_2 5}$

(C) $\frac{2(\log_3 5) + 2}{1 + \log_2 5}$

(D) $\frac{2(\log_3 5) + 1}{1 + \log_2 5}$

$$3^x - 4^y = 5^3 = 125$$

$$(3^{x/2})^2 - (2^y)^2 = 125$$

$$(3^{x/2} - 2^y)(3^{x/2} + 2^y) = 125$$

$$5 \times (3^{x/2} + 2^y) = 125 \quad 25$$

$$3^{x/2} + 2^y = 25$$

$$3^{x/2} - 2^y = 5$$

LOGARITHM & QUADRATIC EQUATION

4. If $x \geq y > 1$ then the maximum value of $\log_x \left(\frac{x}{y} \right) + \log_y \left(\frac{y}{x} \right)$ is equal to

(A) -2

(B) 0

(C) 2

(D) 4

$$\log_x \left(\frac{x}{y} \right) + \log_y \left(\frac{y}{x} \right) = \log_x x - \log_x y + \log_y y - \log_y x = 2 - \left(\log_x y + \frac{1}{\log_y x} \right)$$

$$= 0 \quad \geq 2_{\text{Min}}$$

LOGARITHM & QUADRATIC EQUATION

5. If $\log_2 (\log_8 x) = \log_8 (\log_2 x)$, find the value of $(\log_2 x)^2$.

LOGARITHM & QUADRATIC EQUATION

6. If $10^{\log_a (\log_b (\log_c x))} = 1$ and $10^{(\log_b (\log_c (\log_a x)))} = 1$ then, a is equal to

(A) $\frac{a}{b}$

(B) c^{ab}

(C) ab

(D) $c^{b/c}$

$$x = c^b$$

$$c = a^c$$

$$a^c = c^b$$

$$a = c^{\frac{b}{c}}$$

$$a^3 = 6$$

$$a = 6^{\frac{1}{3}}$$

$$\log_b \log_c \log_a x = 0$$

$$\log_c \log_a x = b^0 = 1$$

$$\log_a x = c^1 = c$$

$$x = a^c$$

LOGARITHM & QUADRATIC EQUATION

7. If $x \in \mathbb{R}$, then number of real solution of the equation $2^x + 2^{-x} = \log_5 24$ is

(A) 0
✓

(B) 1

(C) 2

(D) more than 2

$$2^x + \frac{1}{2^x} = \underbrace{\log_5 24}_{\text{RHS} < 2}$$

≥ 2

No Sol

LOGARITHM & QUADRATIC EQUATION

8. Number of real solution(s) of the equation $9^{\log_3 (\ln x)} = \ln x - (\ln^2 x) + 1$ is equal to

(A) 0

(B) 1

(C) 2

(D) 3 $9^{\log_3(-\frac{1}{2})} = -\frac{1}{2} - \frac{1}{4} + 1$

$$\ln x = -\frac{1}{2}, \ln x = 1$$

$$x = \frac{1}{\sqrt{e}}, x = e$$

$$9^{\log_3(\ln x)} = \ln x - (\ln^2 x) + 1$$

$$2 \log_3 \ln x \quad (2t + 1)(t - 1) = 0$$

$$3 \log_3 (\ln x)^2 \quad t = -\frac{1}{2} \quad t = 1$$

$$(\ln x)^2 = \ln x - (\ln x)^2 + 1$$

$$2(\ln x)^2 - \ln x - 1 = 0$$

$$2t^2 - t - 1 = 0$$

LOGARITHM & QUADRATIC EQUATION

9. Which of the following real numbers is(are) non-positive ?

(A) $\log_{0.3} \left(\frac{\sqrt{5}+2}{\sqrt{5}-2} \right)$ $= \frac{2.2+2}{1.22} = \frac{4.2}{1.22}$

(B) $\log_7 (\sqrt{83} - 9)$ $(9.1 - 9) = .1$
 ? -ve

(C) $\log_{2-\sqrt{3}} (\sqrt{2} + 1)$

(D) $\log_2 \sqrt{9 \cdot \sqrt[3]{27^{\frac{-5}{3}} \cdot 243^{\frac{-7}{5}}}}$

$\log_{0.3} (21)$ $= -ve$

$\log_7 (\sqrt{83} - 9)$ $(9.1 - 9) = .1$

LOGARITHM & QUADRATIC EQUATION

10. Let $x = 4^{\log_2 \sqrt{9^{k-1}+7}}$ and $y = \frac{1}{32^{\log_2 \sqrt[5]{3^{k-1}+1}}}$ and $xy = 4$, then the sum of the cubes of the real value(s) of k is

(A) 1

(B) 5

(C) 8

(D) 9

$$x = 2^{2 \log_2 \sqrt{9^{k-1}+7}}$$

$$= 9^{k-1} + 7$$

$$y = \frac{1}{2^{5 \log_2 (3^{k-1}+1)^{1/5}}} = \frac{1}{(3^{k-1}+1)}$$

$$\frac{1}{(3^{k-1}+1)} \times (3^{2k-2}+7) = 4$$

LOGARITHM & QUADRATIC EQUATION

11.

	Column-I		Column-II
(A)	The value of expression $3^{\sqrt{\log_{27} 8}} - 2^{\sqrt{\log_{32} 243}} - 5^{\sqrt{\log_{625} 81}} + 3^{\sqrt{\log_9 25}} + \sqrt{2}^{\log_2 9} +$ $3^{\log_4 25} - 5^{\log_4 9}, \text{ is less than}$	(P)	2 A) R, S, T B) P
(B)	The value of x satisfying the equation $2^{\log_2 e} \ln 5^{\log_5 7} \log_7 10^{\log_{10} (8x-3)}$ $8x-3 = 13$, is $x=2$	(Q)	C) 3 RST D) P RS
(C)	The number $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right)$ is less than	(R)	4
(D)	Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)(1 + 9^{\log_3 8})^{\log_{65} 5}$ then $(1 + m)$ is divisible by	(S)	5
		(T)	6